

INTRODUCTORY

DIFFERENTIAL EQUATIONS

WITH BOUNDARY VALUE PROBLEMS

THIRD EDITION



ABELL & BRASELTON



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