



SPRINGER OPTIMIZATION  
AND ITS APPLICATIONS

37

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# Introduction to Nonlinear and Global Optimization



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