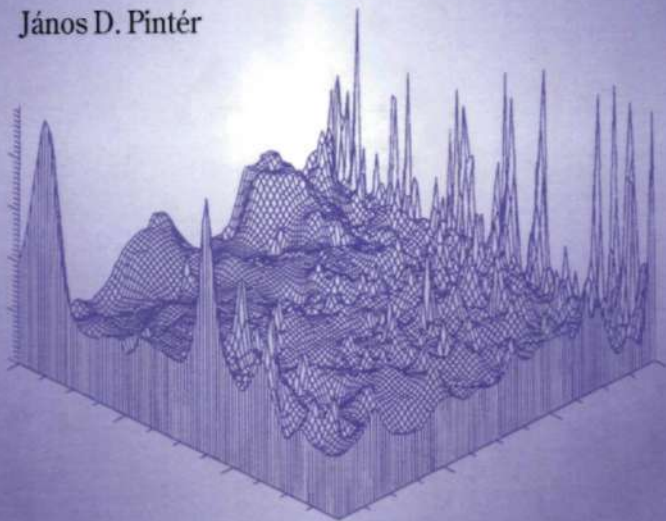


NONCONVEX OPTIMIZATION AND ITS APPLICATIONS

Global Optimization in Action

Continuous and Lipschitz Optimization:
Algorithms, Implementations and Applications

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Kluwer Academic Publishers

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